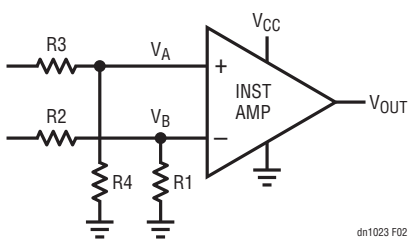# DESIGN NOTES

Precision Matched Resistors Automatically Improve Differential Amplifier CMRR – Here's How
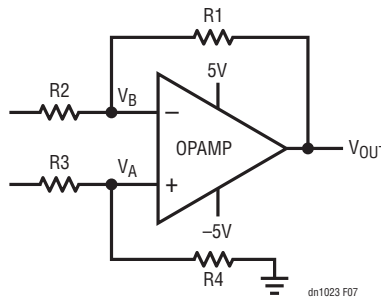
Design Note 1023

Kris Lokere, Tyler Hutchison, Greg Zimmer
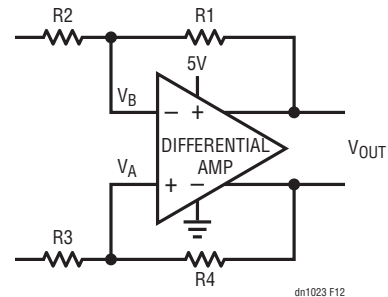
INTRODUCTION

Matched resistors are critical to the performance of a large class of differential circuits such as the following:

Instrumentation Amp

Difference Amp

Differential Amp

Ideally, the resistors in these circuits are chosen such that R1/R2 = R4/R3. Any mismatch between these ratios will contribute a common mode error. The CMRR (common mode rejection ratio) is an important metric in these circuits, as it indicates how much of the unwanted common mode signal will appear in the output. The CMRR due to the resistors in these circuits can be calculated using the following formula:

$$CMRR_{R\_ONLY} \approx \frac{\left(\frac{1}{2}\right)(G+1)}{\Delta R / R}$$

Where $CMRR_{R\_ONLY}$ Is CMRR Due Only to the Resistors (Ideal Amplifier Case), $\Delta R/R$ Is the Resistor Matching Ratio, G is the Nominal Ratio of R1/R2.

For example, using resistors with 1% tolerance (i.e. matched to 2%) in a differential circuit with G = 1, the CMRR will only be 34dB.

LT5400 $\Delta R/R_{CMRR}$

In addition to the guaranteed resistor-to-resistor matching over temperature, the LT5400 includes a new metric called "Matching for CMRR." *Only the LT5400 offers a Matching for CMRR, $(\Delta R/R)_{CMRR}$ specification*. This spec guarantees the contribution of CMRR error due to the resistors, when the LT5400 is used in a difference configuration using the specific resistor pairs of R1/R2 and R4/R3.

The definition of Matching for CMRR is the following:

$$\Delta R / R_{CMRR} = \left(\frac{1}{2}\right)\left(\frac{R1}{R2}\right)\left(\frac{R2}{R1} - \frac{R3}{R4}\right) = \left(\frac{1}{2}\right)\left(1 - \frac{R1R3}{R2R4}\right)$$

Calculating the improved CMRR performance of the LT5400 resistors is easy. Simply replace the resistor matching ratio, (ΔR/R) with the Matching for CMRR specification (ΔR/R$_{CMRR}$):
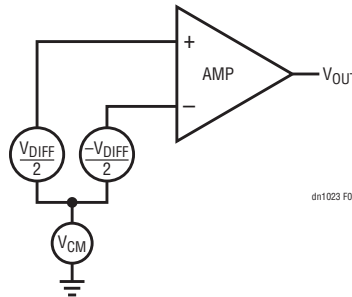
$$CMRR_{R\_ONLY} \approx \frac{\left(\frac{1}{2}\right)(G+1)}{\Delta R / R_{CMRR}}$$

Equation 1
Where ΔR/R$_{CMRR}$ Is the Matching for CMRR specification

For example, the LT5400A offers 0.01% resistor matching and Matching for CMRR of 0.005%. With this Matching for CMRR, the resulting CMRR is 86dB.

CMRR DEFINITION

The amplifier's common mode rejection ratio (CMRR) is the ratio of the differential mode gain to the common mode gain. For these calculations, only common mode and differential mode gain is considered for amplifiers. Thus, an amplifier's output can be determined as: $V_{OUT} = (V_{CM} \bullet A_{CM}) + (V_{DIFF} \bullet A_{DIFF})$

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COMBINING AMPLIFIER AND RESISTOR CMRR

In addition to the CMRR due to the resistors, one must also consider the finite CMRR contribution due to the amplifier. The total CMRR of any of the differential circuits shown above can be calculated using the following formula:

$$CMRR_{TOTAL} \approx \frac{\left(\frac{1}{2}\right)(G+1)}{\left(\frac{1}{CMRR_{amp}}\right)\left(\frac{1}{2}\right)(G+1) + \Delta R / R_{CMRR}}$$

Equation 2
Where CMRR$_{amp}$ Is the Amplifier's CMRR Specification, and CMRR$_{TOTAL}$ Is the Combined CMRR Due to the Resistors and Amplifier.

CMRR CALCULATION

The rest of this design note provides the derivation of CMRR for each differential circuit shown above. The results will show that CMRR can be calculated using Equation 1 and Equation 2 for all three circuits. Examples of calculating CMRR$_{TOTAL}$ (resistor and amplifier) are also provided at the end of each section.

CASE 1: IDEAL INSTRUMENTATION AMP (CMRR$_{amp}$ IS INFINITE)

CMRR Derivation, Step 1: Calculate the Circuit's Common Mode Gain (A$_{CM}$)

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$$V_A = V_{CM}\left(\frac{R4}{R3+R4}\right)$$

$$V_B = V_{CM}\left(\frac{R1}{R1+R2}\right)$$
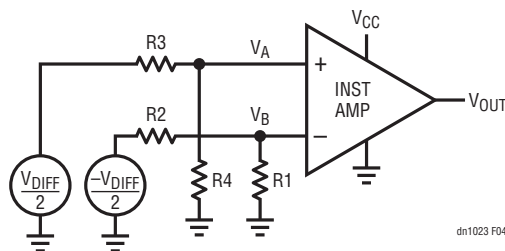
$$V_{OUT} = (V_A - V_B) \bullet G_{amp}$$

Ideal Instrumentation Amplifier, G$_{amp}$ = Amplifier Gain

$$V_{OUT} = \left[V_{CM}\left(\frac{R4}{R3+R4}\right) - V_{CM}\left(\frac{R1}{R1+R2}\right)\right]G_{amp}$$

$$V_{OUT} = V_{CM}G_{amp}\left[\left(\frac{R4}{R3+R4}\right) - \left(\frac{R1}{R1+R2}\right)\right]$$

$$\boxed{\frac{V_{OUT}}{V_{CM}} = G_{amp}\left(\frac{R4}{R3+R4} - \frac{R1}{R1+R2}\right)}$$

CMRR Derivation, Step 2: Calculate the Circuit's Differential Mode Gain (A$_{DIFF}$)

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$$V_A = \frac{V_{DIFF}}{2}\left(\frac{R4}{R3+R4}\right)$$

$$V_B = -\frac{V_{DIFF}}{2}\left(\frac{R1}{R1+R2}\right)$$

$$V_{OUT} = (V_A - V_B) \bullet G_{amp}$$

Ideal Instrumentation Amplifier, G$_{amp}$ = Amplifier Gain

$$V_{OUT} = \left[\left(\frac{V_{DIFF}}{2} \right) \left(\frac{R4}{R3+R4} \right) - \left(-\frac{V_{DIFF}}{2} \right) \left(\frac{R1}{R1+R2} \right) \right] G_{amp}$$

$$V_{OUT} = V_{DIFF} G_{amp} \left(\frac{1}{2} \right) \left(\frac{R4}{R3+R4} + \frac{R1}{R1+R2} \right)$$

$$\boxed{\frac{V_{OUT}}{V_{DIFF}} = G_{amp} \left(\frac{1}{2} \right) \left(\frac{R1}{R1+R2} + \frac{R4}{R3+R4} \right)}$$

CMRR Derivation, Step 3: Calculate the Circuit's CMRR (A_{DIFF}/A_{CM})

$$CMRR_{R\_ONLY} = \frac{G_{amp} \left(\frac{1}{2} \right) \left(\frac{R1}{R1+R2} + \frac{R4}{R3+R4} \right)}{G_{amp} \left(\frac{R4}{R3+R4} - \frac{R1}{R1+R2} \right)}$$

$$= \frac{\left(\frac{1}{2} \right) \left(\frac{R1}{R1+R2} + \frac{R4}{R3+R4} \right)}{\left(\frac{R4}{R3+R4} - \frac{R1}{R1+R2} \right)}$$

Equation 3
This Preliminary Result Will Be Referenced in Other Calculations

$$= \frac{\left(\frac{1}{2} \right) (R1R3 + R1R4 + R1R4 + R2R4)}{(R1R4 + R2R4 - R1R3 - R1R4)}$$

$$= \frac{\left(\frac{1}{2} \right) (2R1R4 + R2R4 + R1R3)}{(R2R4 - R1R3)}$$

$$= \frac{\left(\frac{1}{2} \right) \left(\frac{1}{R2R4} \right) \left(\frac{1}{2} \right) (2R1R4 + R2R4 + R1R3)}{\left(\frac{1}{2} \right) \left(\frac{1}{R2R4} \right) (R2R4 - R1R3)}$$

$$= \frac{\left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \left(\frac{2R1R4}{R2R4} + \frac{R2R4}{R2R4} + \frac{R1R3}{R2R4} \right)}{\left(\frac{1}{2} \right) \left(\frac{R2R4}{R2R4} - \frac{R1R3}{R2R4} \right)}$$

$$CMRR_{R\_ONLY} = \frac{\left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \left(\frac{2R1}{R2} + 1 + \frac{R1R3}{R2R4} \right)}{\left(\frac{1}{2} \right) \left(1 - \frac{R1R3}{R2R4} \right)}$$

4

This equation can be simplified by noting the following:

$$\Delta R / R_{CMRR} = \left(\frac{1}{2}\right)\left(1 - \frac{R1R3}{R2R4}\right) \text{ and } \frac{R1}{R2} \approx \frac{R4}{R3} \approx G \text{ and } \left(\frac{R1R3}{R2R4}\right) \approx 1$$
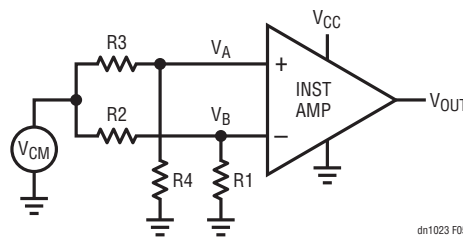
Thus,

$$CMRR_{R\_ONLY} \approx \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(2G+2)}{\left(\frac{1}{2}\right)\left(1 - \frac{R1R3}{R2R4}\right)}$$

$$\boxed{CMRR_{R\_ONLY} \approx \frac{\left(\frac{1}{2}\right)(G+1)}{\Delta R / R_{CMRR}}}$$

CASE 2: NON-IDEAL INSTRUMENTATION AMP (CMRR$_{amp}$ IS FINITE)

CMRR Derivation, Step 1: Calculate the Circuit's Common Mode Gain (A$_{CM}$)

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$$V_{CM}' = (V_A + V_B)/2$$
$$V_{DIFF}' = (V_A - V_B)$$

$$V_A = V_{CM} \bullet \left(\frac{R4}{R3+R4}\right)$$

$$V_B = V_{CM} \bullet \left(\frac{R1}{R1+R2}\right)$$

$$V_{OUT} = (V_{CM}' \bullet A_{CM}') + (V_{DIFF}' \bullet A_{DIFF}')$$

V$_{CM}'$ = Common Mode Voltage Seen at Inputs of Amp
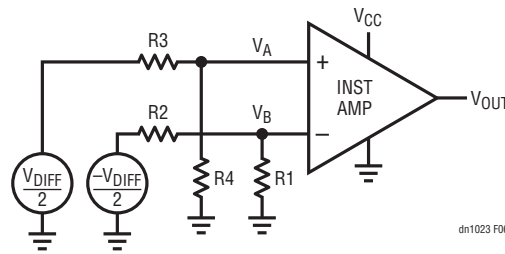**V$_{DIFF}'$ = Differential Voltage Seen at Inputs of Amp**

A$_{CM}'$ = Common Mode Gain of the Amp
A$_{DIFF}'$ = Differential Mode Gain of the Amp

$$V_{OUT} = \frac{1}{2}A_{CM}'\left[V_{CM}\left(\frac{R4}{R3+R4}\right) + V_{CM}\left(\frac{R1}{R1+R2}\right)\right] + A_{DIFF}'\left[V_{CM}\left(\frac{R4}{R3+R4}\right) - V_{CM}\left(\frac{R1}{R1+R2}\right)\right]$$

$$V_{OUT} = V_{CM}\left[\frac{1}{2}A_{CM}'\left[\left(\frac{R4}{R3+R4}\right) + \left(\frac{R1}{R1+R2}\right)\right] + A_{DIFF}'\left[\left(\frac{R4}{R3+R4}\right) - \left(\frac{R1}{R1+R2}\right)\right]\right]$$

$$\boxed{\frac{V_{OUT}}{V_{CM}} = \left[\frac{1}{2}A_{CM}'\left[\left(\frac{R4}{R3+R4}\right) + \left(\frac{R1}{R1+R2}\right)\right] + A_{DIFF}'\left[\left(\frac{R4}{R3+R4}\right) - \left(\frac{R1}{R1+R2}\right)\right]\right]}$$

CMRR Derivation, Step 2: Calculate the Circuit's Differential Mode Gain (A_{DIFF})

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$$V_{CM}{}' = (V_A + V_B)/2$$
$$V_{DIFF}{}' = (V_A - V_B)$$

$V_{CM}{}'$ = Common Mode Voltage Seen at Inputs of Amp
$V_{DIFF}{}'$ = Differential Voltage Seen at Inputs of Amp

$$V_A = \frac{V_{DIFF}}{2} \bullet \left(\frac{R4}{R3+R4} \right)$$

$$V_B = -\frac{V_{DIFF}}{2} \bullet \left(\frac{R1}{R1+R2} \right)$$

$$V_{OUT} = (V_{CM}{}' \bullet A_{CM}{}') + (V_{DIFF}{}' \bullet A_{DIFF}{}')$$

$A_{CM}{}'$ = Common Mode Gain of the Amp
$A_{DIFF}{}'$ = Differential Mode Gain of the Amp

$$V_{OUT} = \frac{1}{2} A_{CM}{}' \left[\frac{V_{DIFF}}{2} \left(\frac{R4}{R3+R4} \right) + \left(-\frac{V_{DIFF}}{2} \right) \left(\frac{R1}{R1+R2} \right) \right]$$

$$+ A_{DIFF}{}' \left[\frac{V_{DIFF}}{2} \left(\frac{R4}{R3+R4} \right) - \left(-\frac{V_{DIFF}}{2} \right) \left(\frac{R1}{R1+R2} \right) \right]$$

$$V_{OUT} = \frac{V_{DIFF}}{2} \left[\frac{1}{2} A_{CM}{}' \left[\left(\frac{R4}{R3+R4} \right) - \left(\frac{R1}{R1+R2} \right) \right] + A_{DIFF}{}' \left[\left(\frac{R4}{R3+R4} \right) + \left(\frac{R1}{R1+R2} \right) \right] \right]$$

$$\frac{V_{OUT}}{V_{DIFF}} = \frac{1}{2} \left[\frac{1}{2} A_{CM}{}' \left[\left(\frac{R4}{R3+R4} \right) - \left(\frac{R1}{R1+R2} \right) \right] + A_{DIFF}{}' \left[\left(\frac{R4}{R3+R4} \right) + \left(\frac{R1}{R1+R2} \right) \right] \right]$$

This equation can be simplified by noting the following:

$$A_{DIFF}{}' \left[\left(\frac{R4}{R3+R4} \right) + \left(\frac{R1}{R1+R2} \right) \right] \gg \frac{1}{2} A_{CM}{}' \left[\left(\frac{R4}{R3+R4} \right) - \left(\frac{R1}{R1+R2} \right) \right]$$

Thus,

$$\boxed{\frac{V_{OUT}}{V_{DIFF}} \approx \frac{1}{2} \left[A_{DIFF}{}' \left[\left(\frac{R4}{R3+R4} \right) + \left(\frac{R1}{R1+R2} \right) \right] \right]}$$

CMRR Derivation, Step 3: Calculate the Circuit's CMRR (A_{DIFF}/A_{CM})

$$CMRR_{TOTAL} \approx \frac{\left(\frac{1}{2}\right)A_{DIFF}'\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]}{\frac{1}{2}A_{CM}'\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]+A_{DIFF}'\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]}$$

Equation 4
This Preliminary Result Will Be Referenced in Other Calculations

$$\approx \frac{\left(\frac{1}{2}\right)\left[(R1R3+R1R4+R1R4+R2R4)\right]}{\left(\frac{1}{2}\right)\left(\frac{1}{CMRR_{amp}}\right)(R1R3+R1R4+R1R4+R2R4)+(R1R4+R2R4-R1R3-R1R4)}$$

$$CMRR_{amp} = \frac{A_{DIFF}'}{A_{CM}'}$$

$$\approx \frac{\left(\frac{1}{2}\right)\left(\frac{1}{R2R4}\right)\left(\frac{1}{2}\right)(R1R3+2R1R4+R2R4)}{\left(\frac{1}{2}\right)\left(\frac{1}{R2R4}\right)\left[\left(\frac{1}{2}\right)\left(\frac{1}{CMRR_{amp}}\right)(R1R3+2R1R4+R2R4)+(R2R4-R1R3)\right]}$$

$$\approx \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{R1R3}{R2R4}+\frac{2R1R4}{R2R4}+\frac{R2R4}{R2R4}\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{CMRR_{amp}}\right)\left(\frac{R1R3}{R2R4}+\frac{2R1R4}{R2R4}+\frac{R2R4}{R2R4}\right)+\left(\frac{1}{2}\right)\left(\frac{R2R4}{R2R4}-\frac{R1R3}{R2R4}\right)}$$

$$\approx \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{R1R3}{R2R4}+\frac{2R1}{R2}+1\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{CMRR_{amp}}\right)\left(\frac{R1R3}{R2R4}+\frac{2R1}{R2}+1\right)+\left(\frac{1}{2}\right)\left(1-\frac{R1R3}{R2R4}\right)}$$

This equation can be simplified by noting the following:

$$\Delta R/R_{CMRR} = \left(\frac{1}{2}\right)\left(1-\frac{R1R3}{R2R4}\right) \text{ and } \frac{R1}{R2} \approx \frac{R4}{R3} \approx G \text{ and } \left(\frac{R1R3}{R2R4}\right) \approx 1$$

Thus,

$$CMRR_{TOTAL} \approx \frac{\left(\frac{1}{2}\right)\left(1+\frac{R1}{R2}\right)}{\left(\frac{1}{CMRR_{amp}}\right)\left(\frac{1}{2}\right)\left(1+\frac{R1}{R2}\right)+\left(\frac{1}{2}\right)\left(1-\frac{R1R3}{R2R4}\right)}$$

$$CMRR_{TOTAL} \approx \frac{\left(\frac{1}{2}\right)(1+G)}{\left(\frac{1}{CMRR_{amp}}\right)\left(\frac{1}{2}\right)(1+G)+\Delta R/R_{CMRR}}$$

Example of CMRR Calculation

Using Linear Technology's LTC2053 Instrument Amplifier, configured for a gain of 1, and the LT5400A-1:

$$\text{CMRR}_{amp(2053)} = 100dB$$

$$G \approx \frac{R1}{R2} \approx \frac{R4}{R3} \approx \frac{10k}{10k} \approx 1$$

$$100dB \rightarrow (1/\text{CMRR}_{amp(2053)}) = 0.001\%$$
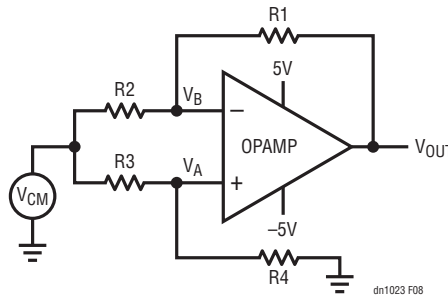
$$\Delta R/R_{CMRR} = 0.005\%$$

Using Equation 2:

$$\text{CMRR}_{TOTAL} \approx \frac{(1)}{\left(\dfrac{1}{\text{CMRR}_{amp}}\right)(1)+\left(\Delta R/R_{CMRR}\right)}$$

$$\approx \frac{1}{0.00001+0.00005}$$

$$\approx 16667$$

$$\text{CMRR}_{TOTAL}(dB) \approx 20\log(16667)$$

$$\approx 84.44dB$$

CASE 3: DIFFERENCE AMP USING AN IDEAL OP AMP (CMRR$_{amp}$ IS INFINITE)

CMRR Derivation, Step 1: Calculate the Circuit's Common Mode Gain (A$_{CM}$)

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$$V_B = V_{CM}\left(\frac{R1}{R1+R2}\right) + V_{OUT}\left(\frac{R2}{R1+R2}\right)$$

$$V_A = V_{CM}\left(\frac{R4}{R3+R4}\right)$$

$$V_A = V_B \qquad\qquad\qquad\qquad\qquad \textcolor{red}{\textbf{Ideal Op Amp}}$$
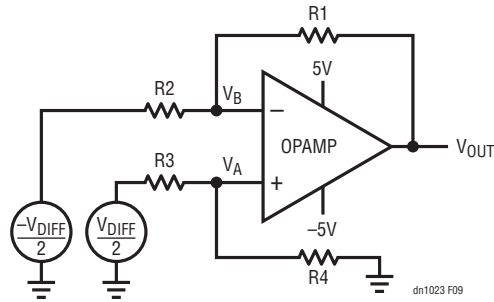
$$V_{CM}\left(\frac{R1}{R1+R2}\right) + V_{OUT}\left(\frac{R2}{R1+R2}\right) = V_{CM}\left(\frac{R4}{R3+R4}\right)$$

$$V_{OUT}\left(\frac{R2}{R1+R2}\right) = V_{CM}\left(\frac{R4}{R3+R4}-\frac{R1}{R1+R2}\right)$$

$$\boxed{\frac{V_{OUT}}{V_{CM}} = \frac{\left(\dfrac{R4}{R3+R4} - \dfrac{R1}{R1+R2}\right)}{\left(\dfrac{R2}{R1+R2}\right)}}$$

CMRR Derivation, Step 2: Calculate the Circuit's Differential Mode Gain (A$_{DIFF}$)

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$$V_A = \frac{V_{DIFF}}{2}\left(\frac{R4}{R3+R4}\right)$$

$$V_B = -\frac{V_{DIFF}}{2}\left(\frac{R1}{R1+R2}\right) + V_{OUT}\left(\frac{R2}{R1+R2}\right)$$

$$V_A = V_B \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \textcolor{maroon}{\textbf{Ideal Op Amp}}$$

$$-\frac{V_{DIFF}}{2}\left(\frac{R1}{R1+R2}\right) + V_{OUT}\left(\frac{R2}{R1+R2}\right) = \frac{V_{DIFF}}{2}\left(\frac{R4}{R3+R4}\right)$$

$$V_{OUT}\left(\frac{R2}{R1+R2}\right) = \left(\frac{V_{DIFF}}{2}\right)\left(\frac{R1}{R1+R2} + \frac{R4}{R3+R4}\right)$$

$$\boxed{\frac{V_{OUT}}{V_{DIFF}} = \frac{\left(\dfrac{1}{2}\right)\left(\dfrac{R1}{R1+R2} + \dfrac{R4}{R3+R4}\right)}{\left(\dfrac{R2}{R1+R2}\right)}}$$

CMRR Derivation, Step 3: Calculate the Circuit's CMRR (A_{DIFF}/A_{CM})

$$CMRR_{R\_ONLY} = \dfrac{\dfrac{\left(\dfrac{1}{2}\right)\left(\dfrac{R1}{R1+R2}+\dfrac{R4}{R3+R4}\right)}{\left(\dfrac{R2}{R1+R2}\right)}}{\dfrac{\left(\dfrac{R4}{R3+R4}-\dfrac{R1}{R1+R2}\right)}{\left(\dfrac{R2}{R1+R2}\right)}}$$
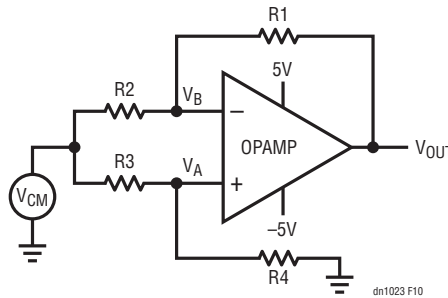
$$= \dfrac{\left(\dfrac{1}{2}\right)\left(\dfrac{R1}{R1+R2}+\dfrac{R4}{R3+R4}\right)}{\left(\dfrac{R4}{R3+R4}-\dfrac{R1}{R1+R2}\right)}$$

This is Equation 3, and we have already shown that Equation 3 can be reduced to Equation 1.

$$CMRR_{R\_ONLY} \approx \dfrac{\left(\dfrac{1}{2}\right)(G+1)}{\Delta R/R_{CMRR}}$$

CASE 4: DIFFERENCE AMP USING A NON-IDEAL OP AMP ($CMRR_{amp}$ IS FINITE)

CMRR Derivation, Step 1: Calculate the Circuit's Common Mode Gain (A_{CM})

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$V_{CM}' = (V_A + V_B)/2$

$V_{DIFF}' = (V_A - V_B)$

$V_A = V_{CM} \bullet \left(\dfrac{R4}{R3+R4}\right)$

$V_B = V_{CM} \bullet \left(\dfrac{R1}{R1+R2}\right) + V_{OUT} \bullet \left(\dfrac{R2}{R1+R2}\right)$

$V_{OUT} = (V_{CM}' \bullet A_{CM}') + (V_{DIFF}' \bullet A_{DIFF}')$

V_{CM}' = Common Mode Voltage Seen at Inputs of Amp
V_{DIFF}' = Differential Voltage Seen at Inputs of Amp

A_{CM}' = Common Mode Gain of the Amp
A_{DIFF}' = Differential Mode Gain of the Amp

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$$V_{OUT} = \frac{1}{2}A_{CM}'\left[V_{CM}\left(\frac{R4}{R3+R4}\right)+V_{CM}\left(\frac{R1}{R1+R2}\right)+V_{OUT}\left(\frac{R2}{R1+R2}\right)\right]$$

$$+A_{DIFF}'\left[V_{CM}\left(\frac{R4}{R3+R4}\right)-V_{CM}\left(\frac{R1}{R1+R2}\right)-V_{OUT}\left(\frac{R2}{R1+R2}\right)\right]$$
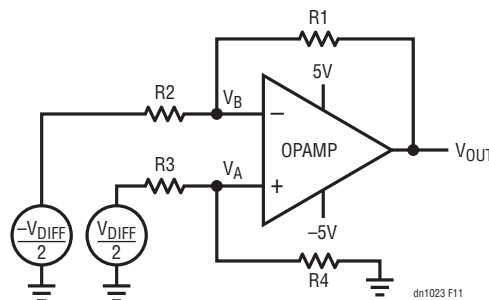
$$V_{OUT} = V_{OUT}\left[\frac{1}{2}A_{CM}'\left(\frac{R2}{R1+R2}\right)-A_{DIFF}'\left(\frac{R2}{R1+R2}\right)\right]$$

$$+V_{CM}\left[\frac{1}{2}A_{CM}'\left[\left(\frac{R1}{R1+R2}\right)+\left(\frac{R4}{R3+R4}\right)\right]+A_{DIFF}'\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]\right]$$

$$V_{OUT}-V_{OUT}\left[\frac{1}{2}A_{CM}'\left(\frac{R2}{R1+R2}\right)-A_{DIFF}'\left(\frac{R2}{R1+R2}\right)\right]$$

$$=V_{CM}\left[\frac{1}{2}A_{CM}'\left[\left(\frac{R1}{R1+R2}\right)+\left(\frac{R4}{R3+R4}\right)\right]+A_{DIFF}'\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]\right]$$

$$V_{OUT}\left[1-\left[\frac{1}{2}A_{CM}'\left(\frac{R2}{R1+R2}\right)-A_{DIFF}'\left(\frac{R1}{R1+R2}\right)\right]\right]$$

$$=V_{CM}\left[\frac{1}{2}A_{CM}'\left[\left(\frac{R1}{R1+R2}\right)+\left(\frac{R4}{R3+R4}\right)\right]+A_{DIFF}'\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]\right]$$

$$\boxed{\frac{V_{OUT}}{V_{CM}}=\frac{\frac{1}{2}A_{CM}'\left[\left(\frac{R1}{R1+R2}\right)+\left(\frac{R4}{R3+R4}\right)\right]+A_{DIFF}'\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]}{1-\left[\frac{1}{2}A_{CM}'\left(\frac{R2}{R1+R2}\right)-\left(A_{DIFF}'\right)\left(\frac{R1}{R1+R2}\right)\right]}}$$

CMRR Derivation, Step 2: Calculate the Circuit's Differential Mode Gain (A_{DIFF})

dn1023 F11

$$V_{CM}'=(V_A+V_B)/2$$
$$V_{DIFF}'=(V_A-V_B)$$

V_{CM}' = Common Mode Voltage Seen at Inputs of Amp
V_{DIFF}' = Differential Voltage Seen at Inputs of Amp

$$V_A = \frac{V_{DIFF}}{2} \bullet \left(\frac{R4}{R3+R4}\right)$$

$$V_B = -\frac{V_{DIFF}}{2} \bullet \left(\frac{R1}{R1+R2}\right) + V_{OUT} \bullet \left(\frac{R2}{R1+R2}\right)$$

$$V_{OUT} = (V_{CM} \bullet A_{CM}{}') + (V_{DIFF} \bullet A_{DIFF}{}')$$

$A_{CM}{}'$ = **Common Mode Gain of the Amp**
$A_{DIFF}{}'$ = **Differential Mode Gain of the Amp**

$$V_{OUT} = \left(\frac{1}{2}A_{CM}{}'\right)\left[\frac{V_{DIFF}}{2}\left(\frac{R4}{R3+R4}\right) + \left(-\frac{V_{DIFF}}{2}\right)\left(\frac{R1}{R1+R2}\right) + V_{OUT}\left(\frac{R2}{R1+R2}\right)\right]$$
$$+ A_{DIFF}{}'\left[\frac{V_{DIFF}}{2}\left(\frac{R4}{R3+R4}\right) - \left(-\frac{V_{DIFF}}{2}\right)\left(\frac{R1}{R1+R2}\right) - V_{OUT}\left(\frac{R2}{R1+R2}\right)\right]$$

$$V_{OUT} = V_{OUT}\left[\frac{1}{2}A_{CM}{}'\left(\frac{R2}{R1+R2}\right) - A_{DIFF}{}'\left(\frac{R2}{R1+R2}\right)\right]$$
$$+ \frac{V_{DIFF}}{2}\left[\frac{1}{2}A_{CM}{}'\left[\left(\frac{R4}{R3+R4}\right) - \left(\frac{R1}{R1+R2}\right)\right] + A_{DIFF}{}'\left[\left(\frac{R4}{R3+R4}\right) + \left(\frac{R1}{R1+R2}\right)\right]\right]$$

$$V_{OUT} - V_{OUT}\left[\frac{1}{2}A_{CM}{}'\left(\frac{R2}{R1+R2}\right) - A_{DIFF}{}'\left(\frac{R2}{R1+R2}\right)\right]$$
$$= \frac{V_{DIFF}}{2}\left[\frac{1}{2}A_{CM}{}'\left[\left(\frac{R4}{R3+R4}\right) - \left(\frac{R1}{R1+R2}\right)\right] + A_{DIFF}{}'\left[\left(\frac{R4}{R3+R4}\right) + \left(\frac{R1}{R1+R2}\right)\right]\right]$$

$$\frac{V_{OUT}}{V_{DIFF}} = \frac{\left(\frac{1}{2}\right)\left[\frac{1}{2}A_{CM}{}'\left[\left(\frac{R4}{R3+R4}\right) - \left(\frac{R1}{R1+R2}\right)\right] + A_{DIFF}{}'\left[\left(\frac{R4}{R3+R4}\right) + \left(\frac{R1}{R1+R2}\right)\right]\right]}{1 - \left[\left(\frac{1}{2}A_{CM}{}'\right)\left(\frac{R2}{R1+R2}\right) - A_{DIFF}{}'\left(\frac{R2}{R1+R2}\right)\right]}$$

This equation can be simplified by noting the following:

$$\left(A_{DIFF}{}'\right)\left[\left(\frac{R4}{R3+R4}\right) + \left(\frac{R1}{R1+R2}\right)\right] \gg \left(\frac{1}{2}A_{CM}{}'\right)\left[\left(\frac{R4}{R3+R4}\right) - \left(\frac{R1}{R1+R2}\right)\right]$$

Thus,

$$\boxed{\frac{V_{OUT}}{V_{DIFF}} \approx \frac{\left(\frac{1}{2}\right)A_{DIFF}{}'\left[\left(\frac{R4}{R3+R4}\right) + \left(\frac{R1}{R1+R2}\right)\right]}{1 - \left[\left(\frac{1}{2}A_{CM}{}'\right)\left(\frac{R2}{R1+R2}\right) - A_{DIFF}{}'\left(\frac{R2}{R1+R2}\right)\right]}}$$

CMRR Derivation, Step 3: Calculate the Circuit's CMRR (A_{DIFF}/A_{CM})

$$CMRR_{TOTAL} \approx \cfrac{\cfrac{\left(\frac{1}{2}\right)A_{DIFF}\cdot\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]}{1-\left[\left(\frac{1}{2}A_{CM}'\right)\left(\frac{R2}{R1+R2}\right)-A_{DIFF}'\left(\frac{R2}{R1+R2}\right)\right]}}{\cfrac{\frac{1}{2}A_{CM}'\left[\left(\frac{R1}{R1+R2}\right)+\left(\frac{R4}{R3+R4}\right)\right]+A_{DIFF}'\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]}{1-\left[\left(\frac{1}{2}A_{CM}'\right)\left(\frac{R2}{R1+R2}\right)-\left(A_{DIFF}'\right)\left(\frac{R1}{R1+R2}\right)\right]}}$$

$$\approx \cfrac{\left(\frac{1}{2}\right)A_{DIFF}\cdot\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]}{\frac{1}{2}A_{CM}'\left[\left(\frac{R1}{R1+R2}\right)+\left(\frac{R4}{R3+R4}\right)\right]+A_{DIFF}'\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]}$$

This is Equation 4, and we have already shown that Equation 4 can be reduced to Equation 2.

$$CMRR_{TOTAL} \approx \cfrac{\left(\frac{1}{2}\right)(1+G)}{\left(\frac{1}{CMRR_{amp}}\right)\left(\frac{1}{2}\right)(1+G)+\Delta R/R_{CMRR}}$$

Example of CMRR Calculation

Using Linear Technology's LT1468 op amp and the LT5400A-3:

$$CMRR_{amp(1468)} = 96dB$$

$$G \approx \frac{R1}{R2} \approx \frac{R4}{R3} \approx \frac{100k}{10k} \approx 10$$

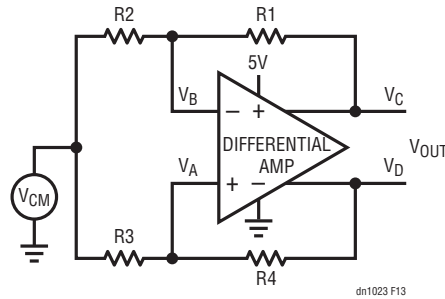$96dB \rightarrow (1/CMRR_{amp(1468)}) = 0.00158\%$

$\Delta R/R_{CMRR} = 0.005\%$

Using Equation 2:

$$CMRR_{TOTAL} \approx \cfrac{(5.5)}{\left(\frac{1}{CMRR_{amp}}\right)(5.5)+\left(\Delta R/R_{CMRR}\right)}$$

$$\approx \frac{5.5}{(0.00005)+(0.0000869)}$$

$$\approx 40175$$

$$CMRR_{TOTAL}(dB) \approx 20\log(40175)$$

$$\approx 92.08dB$$

CASE 5: IDEAL DIFFERENTIAL AMPLIFIER (CMRR$_{amp}$ IS INFINITE)

CMRR Derivation, Step 1: Calculate the Circuit's Common Mode Gain (A$_{CM}$)

dn1023 F13

$$V_{OUT} = V_C - V_D$$

$$\left(\frac{1}{2}\right)V_{OUT} = V_C = -V_D$$

Where V$_{OUT}$ Is the Differential Output Voltage Assuming Balanced Outputs

$$V_A = V_{CM}\left(\frac{R4}{R3+R4}\right) + V_D\left(\frac{R3}{R3+R4}\right)$$

$$V_B = V_{CM}\left(\frac{R1}{R1+R2}\right) + V_C\left(\frac{R2}{R1+R2}\right)$$

$$V_A = V_B$$

Ideal Amplifier

$$V_{CM}\left(\frac{R4}{R3+R4}\right) + V_D\left(\frac{R3}{R3+R4}\right) = V_{CM}\left(\frac{R1}{R1+R2}\right) + V_C\left(\frac{R2}{R1+R2}\right)$$
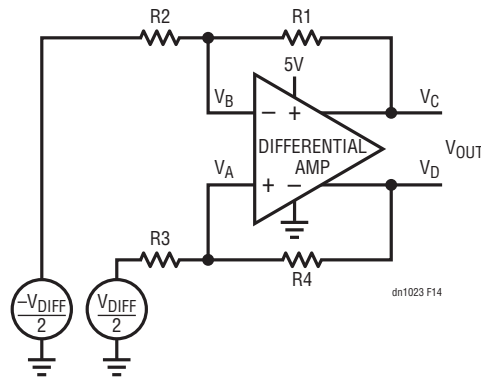
$$V_{CM}\left(\frac{R4}{R3+R4}\right) + (-V_{OUT})\left(\frac{1}{2}\right)\frac{R3}{R3+R4} = V_{CM}\frac{R1}{R1+R2} + V_{OUT}\left(\frac{1}{2}\right)\frac{R2}{R1+R2}$$

$$V_{OUT}\left(\frac{1}{2}\right)\left(\frac{R2}{R1+R2}\right) + V_{OUT}\left(\frac{1}{2}\right)\left(\frac{R3}{R3+R4}\right) = V_{CM}\left(\frac{R4}{R3+R4}\right) - V_{CM}\left(\frac{R1}{R1+R2}\right)$$

$$V_{OUT}\left(\frac{1}{2}\right)\left(\frac{R2}{R1+R2} + \frac{R3}{R3+R4}\right) = V_{CM}\left(\frac{R4}{R3+R4} - \frac{R1}{R1+R2}\right)$$

$$\boxed{\frac{V_{OUT}}{V_{CM}} = \frac{\left(\dfrac{R4}{R3+R4} - \dfrac{R1}{R1+R2}\right)}{\left(\dfrac{1}{2}\right)\left(\dfrac{R2}{R1+R2} + \dfrac{R3}{R3+R4}\right)}}$$

CMRR Derivation, Step 2: Calculate the Circuit's Differential Mode Gain (A$_{DIFF}$)

dn1023 F14

$$V_{OUT} = V_C - V_D$$

$$\left(\frac{1}{2}\right)V_{OUT} = V_C = -V_D$$

Where V$_{OUT}$ Is the Differential Output Voltage Assuming Balanced Outputs

$$V_A = \frac{V_{DIFF}}{2}\left(\frac{R4}{R3+R4}\right) + V_D\left(\frac{R3}{R3+R4}\right)$$

$$V_B = -\frac{V_{DIFF}}{2}\left(\frac{R1}{R1+R2}\right) + V_C\left(\frac{R2}{R1+R2}\right)$$

$$V_A = V_B$$

Ideal Amplifier

$$\frac{V_{DIFF}}{2}\left(\frac{R4}{R3+R4}\right) + \left(-V_{OUT}\right)\left(\frac{1}{2}\right)\left(\frac{R3}{R3+R4}\right) = -\frac{V_{DIFF}}{2}\left(\frac{R1}{R1+R2}\right) + V_{OUT}\left(\frac{1}{2}\right)\left(\frac{R2}{R1+R2}\right)$$

$$V_{OUT}\left(\frac{1}{2}\right)\left(\frac{R2}{R1+R2}\right) + V_{OUT}\left(\frac{1}{2}\right)\left(\frac{R3}{R3+R4}\right) = \frac{V_{DIFF}}{2}\left(\frac{R1}{R1+R2}\right) + \frac{V_{DIFF}}{2}\left(\frac{R4}{R3+R4}\right)$$

$$\boxed{\frac{V_{OUT}}{V_{DIFF}} = \frac{\left(\frac{1}{2}\right)\left(\dfrac{R1}{R1+R2} + \dfrac{R4}{R3+R4}\right)}{\left(\frac{1}{2}\right)\left(\dfrac{R2}{R1+R2} + \dfrac{R3}{R3+R4}\right)}}$$

CMRR Derivation, Step 3: Calculate the Circuit's CMRR (A_{DIFF}/A_{CM})

$$CMRR_{R\_ONLY} = \frac{\dfrac{\left(\dfrac{1}{2}\right)\left(\dfrac{R1}{R1+R2}+\dfrac{R4}{R3+R4}\right)}{\left(\dfrac{1}{2}\right)\left(\dfrac{R2}{R1+R2}+\dfrac{R3}{R3+R4}\right)}}{\dfrac{\left(\dfrac{R4}{R3+R4}-\dfrac{R1}{R1+R2}\right)}{\left(\dfrac{1}{2}\right)\left(\dfrac{R2}{R1+R2}+\dfrac{R3}{R3+R4}\right)}}$$
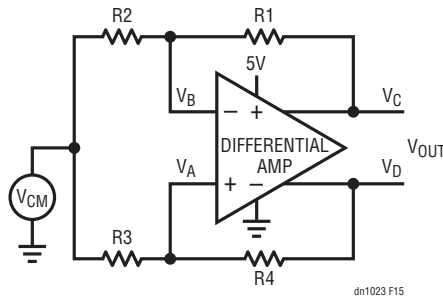
$$CMRR_{R\_ONLY} = \frac{\left(\dfrac{1}{2}\right)\left(\dfrac{R1}{R1+R2}+\dfrac{R4}{R3+R4}\right)}{\left(\dfrac{R4}{R3+R4}-\dfrac{R1}{R1+R2}\right)}$$

This is Equation 3, and we have already shown that Equation 3 can be reduced to Equation 1.

$$CMRR_{R\_ONLY} \approx \frac{\left(\dfrac{1}{2}\right)(G+1)}{\Delta R/R_{CMRR}}$$

CASE 6: NON-IDEAL DIFFERENTIAL AMPLIFIER ($CMRR_{amp}$ IS FINITE)

CMRR Derivation, Step 1: Calculate the Circuit's Common Mode Gain (A_{CM})

dn1023 F15

$V_{OUT} = V_C - V_D$

Where V_{OUT} Is the Differential Output Voltage

$\left(\dfrac{1}{2}\right)V_{OUT} = V_C = -V_D$

Assuming Balanced Outputs

$V_{CM}' = (V_A + V_B)/2$

$V_{DIFF}' = (V_A - V_B)$

V_{CM}' = Common Mode Voltage Seen at Inputs of Amp
V_{DIFF}' = Differential Voltage Seen at Inputs of Amp

$V_A = V_{CM}\left(\dfrac{R4}{R3+R4}\right) + V_D\left(\dfrac{R3}{R3+R4}\right)$

Where A_{CM} Is the Common Mode Input to Differential Output Gain and ADM is the Differential Mode Input to Differential Output Gain

$V_B = V_{CM}\left(\dfrac{R1}{R1+R2}\right) + V_C\left(\dfrac{R2}{R1+R2}\right)$

$V_{OUT} = (V_{CM}' \bullet A_{CM}') + (V_{DIFF}' \bullet A_{DIFF}')$

A_{CM}' = Common Mode Gain of the Amp
A_{DIFF}' = Differential Mode Gain of the Amp

$$V_{OUT} = \frac{1}{2}A_{CM}'\left[V_{CM}\left(\frac{R4}{R3+R4}\right)+\left(-\frac{V_{OUT}}{2}\right)\left(\frac{R3}{R3+R4}\right)+V_{CM}\left(\frac{R1}{R1+R2}\right)+\frac{V_{OUT}}{2}\left(\frac{R2}{R1+R2}\right)\right]$$

$$+A_{DIFF}'\left[V_{CM}\left(\frac{R4}{R3+R4}\right)+\left(-\frac{V_{OUT}}{2}\right)\left(\frac{R3}{R3+R4}\right)-V_{CM}\left(\frac{R1}{R1+R2}\right)-\frac{V_{OUT}}{2}\left(\frac{R2}{R1+R2}\right)\right]$$

$$V_{OUT} = V_{CM}\left[\frac{1}{2}A_{CM}'\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]+A_{DIFF}'\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]\right]$$

$$-\frac{V_{OUT}}{2}\left[\frac{1}{2}A_{CM}'\left[\left(\frac{R3}{R3+R4}\right)-\left(\frac{R2}{R1+R2}\right)\right]+A_{DIFF}'\left[\left(\frac{R3}{R3+R4}\right)+\left(\frac{R2}{R1+R2}\right)\right]\right]$$

$$V_{OUT}+\frac{V_{OUT}}{2}\left[\frac{1}{2}A_{CM}'\left[\left(\frac{R3}{R3+R4}\right)-\left(\frac{R2}{R1+R2}\right)\right]+A_{DIFF}'\left[\left(\frac{R3}{R3+R4}\right)+\left(\frac{R2}{R1+R2}\right)\right]\right]$$

$$=V_{CM}\left[\frac{1}{2}A_{CM}'\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]+A_{DIFF}'\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]\right]$$

$$V_{OUT}\left(1+\left(\frac{1}{2}\right)\left[\frac{1}{2}A_{CM}'\left[\left(\frac{R3}{R3+R4}\right)-\left(\frac{R2}{R1+R2}\right)\right]+A_{DIFF}'\left[\left(\frac{R3}{R3+R4}\right)+\left(\frac{R2}{R1+R2}\right)\right]\right]\right)$$

$$=V_{CM}\left[\frac{1}{2}A_{CM}'\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]+A_{DIFF}'\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]\right]$$

$$\frac{V_{OUT}}{V_{CM}}=\frac{\frac{1}{2}A_{CM}'\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]+A_{DIFF}'\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]}{1+\left(\frac{1}{2}\right)\left[\frac{1}{2}A_{CM}'\left[\left(\frac{R3}{R3+R4}\right)-\left(\frac{R2}{R1+R2}\right)\right]+A_{DIFF}'\left[\left(\frac{R3}{R3+R4}\right)+\left(\frac{R2}{R1+R2}\right)\right]\right]}$$
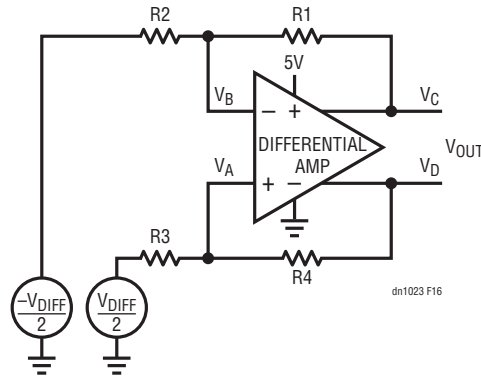
This equation can be simplified by noting the following:

$$\left(A_{DIFF}'\right)\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]>>\left(\frac{1}{2}A_{CM}'\right)\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]$$

Thus,

$$\frac{V_{OUT}}{V_{CM}}\approx\frac{\frac{1}{2}A_{CM}'\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]+A_{DIFF}'\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]}{1+\left(\frac{1}{2}\right)A_{DIFF}'\left[\left(\frac{R3}{R3+R4}\right)+\left(\frac{R2}{R1+R2}\right)\right]}$$

CMRR Derivation, Step 2: Calculate the Circuit's Differential Mode Gain (A_{DIFF})

$V_{OUT} = V_C - V_D$

Where V_{OUT} Is the Differential Output Voltage

$\left(\dfrac{1}{2}\right)V_{OUT} = V_C = -V_D$

Assuming Balanced Outputs

$V_{CM}{}' = (V_A + V_B)/2$

$V_{DIFF}{}' = (V_A - V_B)$

$V_{CM}{}'$ = Common Mode Voltage Seen at Inputs of Amp
$V_{DIFF}{}'$ = Differential Voltage Seen at Inputs of Amp

$V_A = \dfrac{V_{DIFF}}{2}\left(\dfrac{R4}{R3+R4}\right) + V_D\left(\dfrac{R3}{R3+R4}\right)$

Where A_{CM} Is the Common Mode Input to Differential Output Gain and ADM is the Differential Mode Input to Differential Output Gain

$V_B = -\dfrac{V_{DIFF}}{2}\left(\dfrac{R1}{R1+R2}\right) + V_C\left(\dfrac{R2}{R1+R2}\right)$

$V_{OUT} = (V_{CM}{}' \bullet A_{CM}{}') + (V_{DIFF}{}' \bullet A_{DIFF}{}')$

$A_{CM}{}'$ = Common Mode Gain of the Amp
$A_{DIFF}{}'$ = Differential Mode Gain of the Amp

$V_{OUT} = \dfrac{1}{2}A_{CM}{}'\left[\dfrac{V_{DIFF}}{2}\left(\dfrac{R4}{R3+R4}\right) + (-V_{OUT})\left(\dfrac{1}{2}\right)\left(\dfrac{R3}{R3+R4}\right) + \left(-\dfrac{V_{DIFF}}{2}\right)\left(\dfrac{R1}{R1+R2}\right) + V_{OUT}\left(\dfrac{1}{2}\right)\left(\dfrac{R2}{R1+R2}\right)\right]$

$\qquad + A_{DIFF}{}'\left[\dfrac{V_{DIFF}}{2}\left(\dfrac{R4}{R3+R4}\right) + (-V_{OUT})\left(\dfrac{1}{2}\right)\left(\dfrac{R3}{R3+R4}\right) - \left(-\dfrac{V_{DIFF}}{2}\right)\left(\dfrac{R1}{R1+R2}\right) - V_{OUT}\left(\dfrac{1}{2}\right)\left(\dfrac{R2}{R1+R2}\right)\right]$

$V_{OUT} = \dfrac{V_{DIFF}}{2}\left[\dfrac{1}{2}A_{CM}{}'\left[\left(\dfrac{R4}{R3+R4}\right) - \left(\dfrac{R1}{R1+R2}\right)\right] + A_{DIFF}{}'\left[\left(\dfrac{R4}{R3+R4}\right) + \left(\dfrac{R1}{R1+R2}\right)\right]\right]$

$\qquad - V_{OUT}\left(\dfrac{1}{2}\right)\left[\dfrac{1}{2}A_{CM}{}'\left[\left(\dfrac{R3}{R3+R4}\right) - \left(\dfrac{R2}{R1+R2}\right)\right] + A_{DIFF}{}'\left[\left(\dfrac{R3}{R3+R4}\right) + \left(\dfrac{R2}{R1+R2}\right)\right]\right]$

$V_{OUT} + \left(\dfrac{1}{2}\right)V_{OUT}\left[\dfrac{1}{2}A_{CM}{}'\left[\left(\dfrac{R3}{R3+R4}\right) - \left(\dfrac{R2}{R1+R2}\right)\right] + A_{DIFF}{}'\left[\left(\dfrac{R3}{R3+R4}\right) + \left(\dfrac{R2}{R1+R2}\right)\right]\right]$

$\qquad = \dfrac{V_{DIFF}}{2}\left[\dfrac{1}{2}A_{CM}{}'\left[\left(\dfrac{R4}{R3+R4}\right) - \left(\dfrac{R1}{R1+R2}\right)\right] + A_{DIFF}{}'\left[\left(\dfrac{R4}{R3+R4}\right) + \left(\dfrac{R1}{R1+R2}\right)\right]\right]$

$$V_{OUT}\left(1+\left(\frac{1}{2}\right)\left[\frac{1}{2}A_{CM}\cdot\left[\left(\frac{R3}{R3+R4}\right)-\left(\frac{R2}{R1+R2}\right)\right]+A_{DIFF}\cdot\left[\left(\frac{R3}{R3+R4}\right)+\left(\frac{R2}{R1+R2}\right)\right]\right]\right)$$

$$=\frac{V_{DIFF}}{2}\left[\frac{1}{2}A_{CM}\cdot\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]+A_{DIFF}\cdot\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]\right]$$

$$\frac{V_{OUT}}{V_{DIFF}}=\frac{\left(\frac{1}{2}\right)\left[\frac{1}{2}A_{CM}\cdot\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]+A_{DIFF}\cdot\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]\right]}{1+\left(\frac{1}{2}\right)\left[\frac{1}{2}A_{CM}\cdot\left[\left(\frac{R3}{R3+R4}\right)-\left(\frac{R2}{R1+R2}\right)\right]+A_{DIFF}\cdot\left[\left(\frac{R3}{R3+R4}\right)+\left(\frac{R2}{R1+R2}\right)\right]\right]}$$

This equation can be simplified by noting the following:

$$A_{DIFF}\cdot\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]\gg\frac{1}{2}A_{CM}\cdot\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]$$

and

$$A_{DIFF}\cdot\left[\left(\frac{R3}{R3+R4}\right)+\left(\frac{R2}{R1+R2}\right)\right]\gg\frac{1}{2}A_{CM}\cdot\left[\left(\frac{R3}{R3+R4}\right)-\left(\frac{R2}{R1+R2}\right)\right]$$

$$\boxed{\frac{V_{OUT}}{V_{DIFF}}\approx\frac{\left(\frac{1}{2}\right)\left[A_{DIFF}\cdot\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]\right]}{1+\left(\frac{1}{2}\right)A_{DIFF}\cdot\left[\left(\frac{R3}{R3+R4}\right)+\left(\frac{R2}{R1+R2}\right)\right]}}$$

CMRR Derivation, Step 3: Calculate the Circuit's CMRR (A_{DIFF}/A_{CM})

$$CMRR_{TOTAL}\approx\frac{\dfrac{\left(\frac{1}{2}\right)\left[A_{DIFF}\cdot\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]\right]}{1+\left(\frac{1}{2}\right)A_{DIFF}\cdot\left[\left(\frac{R3}{R3+R4}\right)+\left(\frac{R2}{R1+R2}\right)\right]}}{\dfrac{\frac{1}{2}A_{CM}\cdot\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]+A_{DIFF}\cdot\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]}{1+\left(\frac{1}{2}\right)A_{DIFF}\cdot\left[\left(\frac{R3}{R3+R4}\right)+\left(\frac{R2}{R1+R2}\right)\right]}}$$

$$\approx\frac{\left(\frac{1}{2}\right)\left[A_{DIFF}\cdot\left[\left(\frac{R4}{R3+R4}\right)+\left(\frac{R1}{R1+R2}\right)\right]\right]}{\frac{1}{2}A_{CM}\cdot\left[\left(\frac{R1}{R1+R2}\right)+\left(\frac{R4}{R3+R4}\right)\right]+A_{DIFF}\cdot\left[\left(\frac{R4}{R3+R4}\right)-\left(\frac{R1}{R1+R2}\right)\right]}$$

This is Equation 4, and we have already shown that Equation 4 can be reduced to Equation 2.

$$CMRR_{TOTAL} \approx \frac{\left(\frac{1}{2}\right)(1+G)}{\left(\frac{1}{CMRR_{amp}}\right)\left(\frac{1}{2}\right)(1+G)+\Delta R/R_{CMRR}}$$

Example of CMRR Calculation

Using Linear Technology's LTC6362 Differential Amplifier and the LT5400A-6:

$$CMRR_{amp(6362)} = 70dB$$

$$G \approx \frac{R1}{R2} \approx \frac{R4}{R3} \approx \frac{5k}{1k} \approx 5$$

70dB ➜ (1/CMRR_{amp(6362)})=0.0316%

$\Delta R/R_{CMRR}$ = 0.005%

Using Equation 2:

$$CMRR_{TOTAL} \approx \frac{(3)}{\left(\frac{1}{CMRR_{amp}}\right)(3)+\left(\Delta R/R_{CMRR}\right)}$$

$$\approx \frac{3}{(0.00005)+(3 \bullet 0.000316)}$$

$$\approx 3006$$

$$CMRR_{TOTAL}(dB) \approx 20log(3006)$$
$$\approx 69.56dB$$

For applications help,
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